Paper Reference(s)

6681/01

Edexcel GCE

Mechanics M5

Advanced/Advanced Subsidiary

Monday 24 June 2013 − Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P41823A

1. Solve the differential equation

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} - 2\mathbf{r} = \mathbf{0}$$

given that when $t = 0$, $\mathbf{r}.\mathbf{j} = 0$ and $\mathbf{r} \times \mathbf{j} = \mathbf{i} + \mathbf{k}$.

(7)

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2. A uniform square lamina S has side $2a$. The radius of gyration of S about an axis through a vertex, perpendicular to S, is k.

(*a*) Show that $k^2 = \dfrac{8a^2}{3}$.

(4)

The lamina S is free to rotate in a vertical plane about a fixed smooth horizontal axis which is perpendicular to S and passes through a vertex.

(*b*) By writing down an equation of rotational motion for S, find the period of small oscillations of S about its position of stable equilibrium.

(5)

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3. A raindrop falls vertically under gravity through a stationary cloud. At time $t = 0$, the raindrop is at rest and has mass m_0. As the raindrop falls, water condenses onto it from the cloud so that the mass of the raindrop increases at a constant rate c. At time t, the mass of the raindrop is m and the speed of the raindrop is v. The resistance to the motion of the raindrop has magnitude mkv, where k is a constant. Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} + v\left(k + \frac{c}{m_0 + ct}\right) = g$$

(7)

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4. Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act on a rigid body. The forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act through the points with position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ respectively.

$\mathbf{r}_1 = (-2\mathbf{i} + 3\mathbf{j})$ m, $\quad\quad \mathbf{F}_1 = (3\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ N

$\mathbf{r}_2 = (3\mathbf{i} + 2\mathbf{k})$ m, $\quad\quad \mathbf{F}_2 = (-2\mathbf{i} + \mathbf{j} - \mathbf{k})$ N

Given that the system $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ is in equilibrium,

(*a*) find $\mathbf{F}_3$,

(2)

(*b*) find a vector equation of the line of action of $\mathbf{F}_3$, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$.

(5)

The force $\mathbf{F}_3$ is replaced by a force $\mathbf{F}_4$ acting through the point with position vector $(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$ m. The system $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_4$ is equivalent to a single force $(3\mathbf{i} + \mathbf{j} + \mathbf{k})$ N acting through the point with position vector $(\mathbf{i} + \mathbf{j} + \mathbf{k})$ m together with a couple.

(*c*) Find the magnitude of this couple.

(8)

5.

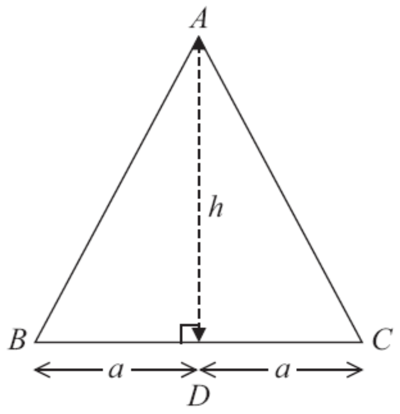

Figure 1

A uniform triangular lamina ABC, of mass M, has $AB = AC$ and $BC = 2a$. The mid-point of BC is D and $AD = h$, as shown in Figure 1.

Show, using integration, that the moment of inertia of the lamina about an axis through A, perpendicular to the plane of the lamina, is

$$\frac{M}{6}(a^2 + 3h^2)$$

[*You may assume without proof that the moment of inertia of a uniform rod, of length 2l and mass m, about an axis through its midpoint and perpendicular to the rod, is* $\frac{1}{3}ml^2$.]

(10)

6.

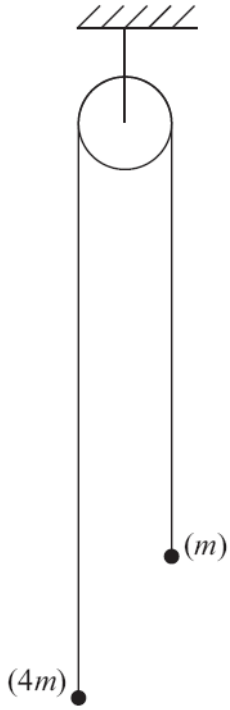

Figure 2

A light inextensible string has a particle of mass m attached to one end and a particle of mass $4m$ attached to the other end. The string passes over a rough pulley which is modelled as a uniform circular disc of radius a and mass $2m$, as shown in Figure 2.

The pulley can rotate in a vertical plane about a fixed horizontal axis which passes through the centre of the pulley and is perpendicular to the plane of the pulley. As the pulley rotates, a frictional couple of constant magnitude $2mga$ acts on it.

The system is held with the string vertical and taut on each side of the pulley and released from rest. Given that the string does not slip on the pulley, find the initial angular acceleration of the pulley.

(10)

7. A uniform circular disc, of radius r and mass m, is free to rotate in a vertical plane about a fixed smooth horizontal axis. This axis is perpendicular to the plane of the disc and passes through a point A on the circumference of the disc. The disc is held with AB horizontal, where AB is a diameter of the disc, and released from rest.

(*a*) Find the magnitude of

(i) the horizontal component,

(ii) the vertical component

of the force exerted on the disc by the axis immediately after the disc is released.

(11)

When AB is vertical the disc is instantaneously brought to rest by a horizontal impulse which acts in the plane of the disc and is applied to the disc at B.

(*b*) Find the magnitude of the impulse.

(6)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	G.S. is $\mathbf{r} = \mathbf{A}e^{2t}$	B1
	$t = 0$: $\mathbf{A}.\mathbf{j} = 0 \Rightarrow \mathbf{A} = p\mathbf{i} + r\mathbf{k}$	M1 A1
	$(p\mathbf{i} + r\mathbf{k}) \times \mathbf{j} = \mathbf{i} + \mathbf{k}$	M1 A1
	$-r\mathbf{i} + p\mathbf{k} = \mathbf{i} + \mathbf{k} \Rightarrow r = -1; p = 1$	M1
	$\mathbf{r} = (\mathbf{i} - \mathbf{k})e^{2t}$	A1
		(7)
		[7]
Notes for Question 1		
	B1 for $\mathbf{r} = \mathbf{A}e^{2t}$ oe. First M1 for *use* of initial conditions $t = 0$, $\mathbf{r}.\mathbf{j} = 0$. (M0 if no explicit **r** expression to sub into) First A1 for $\mathbf{A} = p\mathbf{i} + r\mathbf{k}$ (or $q = 0$) Second M1 for attempt at cross-product **r** x **j** when $t = 0$ (M0 if no explicit **r** expression to sub into) Second A1 for ($-r$ **i** + p **k**) Third M1 for using the second condition to find values for p and r. Third A1 for a correct answer. N.B. All marks available apart from the final A1 if unsound work seen e.g. logs of vectors, <u>provided that logs are removed at the start to give an explicit expression for **r** which can be evaluated in order to find the value of the constant.</u>	

Question Number	Scheme	Marks
2.		
(a)	$I_G = \frac{1}{3}ma^2 + \frac{1}{3}ma^2 = \frac{2}{3}ma^2$ (perp axes)	M1 A1
	$I_G = \frac{2}{3}ma^2 + m(a\sqrt{2})^2 = \frac{8}{3}ma^2$ i.e. $k^2 = \frac{8}{3}a^2$ **	M1 A1
		(4)
(b)	$-mga\sqrt{2}\sin\theta = \frac{8}{3}ma^2\ddot{\theta}$	M1 A1
	$\ddot{\theta} = -\frac{3g\sqrt{2}}{8a}\theta$, for small θ	DM1
	$T = 2\pi\sqrt{\frac{8a}{3g\sqrt{2}}}$	M1 A1
		(5)
		[9]
Notes for Question 2		
2(a)	First M1 for use of perpendicular axes rule with appropriate no. of terms First A1 for correct expression for I_G (or from formulae sheet) Second M1 for use of parallel axes rule to obtain I_B Second A1 for PRINTED ANSWER. Alternative, using result(s) on formulae sheet: First M1A1 $I_{AB} = I_{BC} = 4/3ma^2$ (from formulae sheet) Second M1 for use of perpendicular axes rule with appropriate no. of terms $I_B = I_{AB} + I_{BC} = 8/3ma^2$	
2(b)	First M1 for moments equation (dim correct and mg resolved) First A1 for correct equation Second M1 dependent for use of $\sin\theta = \theta$ and SHM equation Third M1 for use of $T = 2\pi/\omega$ (only if proper SHM equn with -) Third A1 for answer.	

Question Number	Scheme	Marks
3.	$\frac{\mathrm{d}m}{\mathrm{d}t} = c \Rightarrow m = m_0 + ct$	
	$(m+\delta m)(v+\delta v) - mv = (mg - mkv)\delta t$	M1 A1
	$m\delta v + v\delta m = (mg - mkv)\delta t$	M1 A2
	$\frac{\mathrm{d}v}{\mathrm{d}t} + kv + \frac{v}{m}\frac{\mathrm{d}m}{\mathrm{d}t} = g$	
	$\frac{\mathrm{d}v}{\mathrm{d}t} + v\left(k + \frac{c}{m_0 + ct}\right) = g$ **	DM1 A1
		(7)
		[7]

Notes for Question 3

First M1 for $\frac{\mathrm{d}m}{\mathrm{d}t} = c$ and integrating

First A1 for $m = m_0 + ct$

Second M1 for impulse-momentum equation (correct number of terms, excluding any $\delta m \delta v$ or $\delta m \delta t$ terms)

Second and third A1: (-1 each error)

OR:

Second M1 for $mg - mkv = \frac{\mathrm{d}}{\mathrm{d}t}(mv)$

Second and third A1 for $mg - mkv = v\frac{\mathrm{d}m}{\mathrm{d}t} + m\frac{\mathrm{d}v}{\mathrm{d}t}$ (-1 each error)

Third M1, dependent on second M1, for sub. for m .

Third A1 for PRINTED ANSWER

Question Number	Scheme	Marks
4.		
(a)	$(3\mathbf{i} - 2\mathbf{j} + \mathbf{k}) + (-2\mathbf{i} + \mathbf{j} - \mathbf{k}) + \mathbf{F}_3 = \mathbf{0}$	M1
	$\mathbf{F}_3 = (-\mathbf{i} + \mathbf{j})$ (N)	A1
		(2)
(b)	$(-2\mathbf{i} + 3\mathbf{j}) \times (3\mathbf{i} - 2\mathbf{j} + \mathbf{k}) + (3\mathbf{i} + 2\mathbf{k}) \times (-2\mathbf{i} + \mathbf{j} - \mathbf{k}) + \mathbf{r} \times (-\mathbf{i} + \mathbf{j}) = \mathbf{0}$	M1
	$(3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}) + (-2\mathbf{i} - \mathbf{j} + 3\mathbf{k}) + (-z\mathbf{i} - z\mathbf{j} + (x+y)\mathbf{k}) = \mathbf{0}$	A2,1,0
	$1 - z = 0, \quad 1 - z = 0, \quad -2 + x + y = 0$	M1
	$\mathbf{r} = (2\mathbf{i} + \mathbf{k}) + t(-\mathbf{i} + \mathbf{j})$ is a solution	A1
	OR Use Concurrency Principle	
		(5)
(c)	$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_4 = (3\mathbf{i} + \mathbf{j} + \mathbf{k}) \Rightarrow \mathbf{F}_4 = (2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$	M1 A1
	$\mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 + (\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) \times (2\mathbf{i} + 2\mathbf{j} + \mathbf{k}) = (\mathbf{i} + \mathbf{j} + \mathbf{k}) \times (3\mathbf{i} + \mathbf{j} +$	M1
	$(3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}) + (-2\mathbf{i} - \mathbf{j} + 3\mathbf{k}) + (-8\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}) = (2\mathbf{j} - 2\mathbf{k}) + \mathbf{G}$	A2,1,0 **ft on a**
	$\mathbf{G} = (-7\mathbf{i} + 4\mathbf{j} + 6\mathbf{k})$	A1
	$\lvert\mathbf{G}\rvert = \sqrt{(-7)^2 + 4^2 + 6^2} = \sqrt{101}$ (Nm)	M1 A1
		(8)
		[15]

Notes for Question 4

4(a)	M1 for $\Sigma\mathbf{F}_i = \mathbf{0}$ A1 for **(-i + j)**	
4(b)	**In (b) condone consistent use of F x r.** First M1 for M(O) or M(P_1) or M(P_2) (must be using correct forces) First A1 and Second A1 -1each product. Second M1 for changing equation to $\mathbf{r} = \mathbf{a} + \lambda\mathbf{F}_3$ Third A1 for any correct equation. **OR:** Use Concurrency Principle First M1 for $\mathbf{r}_1 + s\mathbf{F}_1 = \mathbf{r}_2 + t\mathbf{F}_2$ First A1 for $s = 1$ or $t = 1$ Second A1 for $\mathbf{i} + \mathbf{j} + \mathbf{k}$ Second M1 for changing equation to $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ Third A1 for any correct equation.	
4(c)	First M1 $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_4 = (3\mathbf{i} + \mathbf{j} + \mathbf{k})$ First A1 for $\mathbf{F}_4 = (2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ Second M1 for $(\mathbf{r}_1 \text{ x } \mathbf{F}_1) + (\mathbf{r}_2 \text{ x } \mathbf{F}_2) + (\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) \text{ x } (\text{their } \mathbf{F}_4) = (\mathbf{i} + \mathbf{j} + \mathbf{k}) \text{ x } (3\mathbf{i} + \mathbf{j}$ Second A1**ft** and Third A1**ft** for correct equation with products evaluated (-1 ee) Fourth A1 for $\mathbf{G} = (-7\mathbf{i} + 4\mathbf{j} + 6\mathbf{k})$ Third M1 for $\lvert\mathbf{G}\rvert = \sqrt{(-7)^2 + 4^2 + 6^2}$ Fifth A1 for √101 oe (2 or more SF)	

Question Number	Scheme	Marks
5.	Taking strips parallel to *BC*:	
	$l_x = \dfrac{2ax}{h}$	M1 A1
	$\delta m = \dfrac{2ax}{h}.\dfrac{M\delta x}{ah} = \dfrac{2Mx\delta x}{h^2}$	M1 A1
	$\delta I = \dfrac{1}{3}\delta m(\dfrac{ax}{h})^2 + \delta m.x^2$	M1 A1A1
	$= \dfrac{2M}{3h^4}(a^2+3h^2)x^3\delta x$	A1
	$I = \dfrac{2M}{3h^4}(a^2+3h^2)\int_0^h x^3 \mathrm{d}x$	DM1
	$= \dfrac{2M}{3h^4}(a^2+3h^2)\left[\dfrac{x^4}{4}\right]_0^h$	
	$= \dfrac{M}{6}(a^2+3h^2)$ **	A1
		(10)
		[10]
Notes for Question 5		
	First M1 for attempt to find length of strip in terms of *x, h* and *a*, using similar triangles or equivalent (must be dim correct) First A1 for a correct expression Second M1 for attempt to find mass of strip in terms of *x, h, M* and δx (must be dim correct) (this mark is not available if ρ is not found) Second A1 for a correct expression. Third M1 for use of parallel axes rule on strip Third A1 for $\frac{1}{12}\delta m l_x^2$ term Fourth A1 for $\delta m x^2$ term Fifth A1 for a correct expression in *a, x, h, M* and δx Fourth M1 dependent on Third M1, for integrating their δI (which must have a δx) Sixth A1 for PRINTED ANSWER N.B. Third M1 They may use perpendicular axes on *whole* triangle, with same working. (N.B. if ρ is used but never eliminated, can score max M1A1M0A0M1A1A1A0M1A0)	

Question Number	Scheme	Marks
6.	$4mg - T_1 = 4mf$	M1 A1
	$T_2 - mg = mf$	M1 A1
	$(T_1 - T_2)a - 2mga = \frac{1}{2}2ma^2.\frac{f}{a}$	M1 A3
	$4mg - mg - 2mg = 6mf \Rightarrow f = \frac{g}{6}$	DM1
	ang accln = $\frac{g}{6a}$	A1
		(10)
		[10]

Notes for Question 6

First M1 for equation of motion for $4m$

First A1 for correct equation using either f or α

Second M1 for equation of motion for m

Second A1 for correct equation using either f or α

Third M1 for equation of motion for pulley

A3 for a correct equation using either f or α

Fourth M1, dependent on previous three M's, for producing an equation in α and g only.

Sixth A1 for answer

S.C. M4A5 'whole system' equation:

$$(4mg - mg - 2mg)a = (4ma^2 + ma^2 + ma^2)\alpha$$

Question Number	Scheme	Marks
7.		
(a)	$I_A = \tfrac{1}{2}mr^2 + mr^2 = \dfrac{3mr^2}{2}$	M1 A1
	$\rightarrow \qquad X = mr\dot{\theta}^2 = 0$	M1 A1A1
	$\downarrow \qquad mg - Y = mr\ddot{\theta}$	M1 A1
	$M(A) \qquad mgr = \dfrac{3mr^2}{2}\ddot{\theta}$	M1 A1
	$Y = \tfrac{1}{3}mg$	DM1 A1
		(11)
(b)	$\dfrac{1}{2}\dfrac{3mr^2}{2}\omega^2 = mgr$	M1 A1
	$I.2r = \dfrac{3mr^2}{2}\omega$	M1 A1
	$I = \dfrac{m}{2}\sqrt{3gr}$	DM1 A1
		(6)
		[17]

Notes for Question 7

7(a)	First M1 for use of parallel axes rule First A1 for correct expression Second M1 for resolving horizontally (usual rules) Second A1 for a correct equation Third A1 for 0 Third M1 for resolving vertically (usual rules) Fourth A1 for a correct equation Fourth M1 for moments about A (usual rules) Fifth A1 for a correct equation Fifth M1, dependent on previous two M marks, for solving for Y. A1 for answer	
7(b)	First M1 for energy equation First A1 for a correct equation Second M1 for angular impulse-momentum equation Second A1 for a correct equation Third M1, dependent on previous M's, for solving for I Third A1 for the answer	